THE SHEAR STRENGTH OF GRANULAR MATERIALS WITH DISPERSED AND NON-DISPERSED OVERSIZED PARTICLES

by

Leanna Seminsky

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This thesis was presented

by

Leanna Seminsky

It was defended on

April 17, 2013

and approved by

Jorge D. Abad, Ph. D., Assistant Professor, Department of Civil and Environmental Engineering

Calixto I. Garcia, Ph. D., Professor, Department of Mechanical Engineering and Materials Science

Luis E. Vallejo, Ph. D., Professor, Department of Civil and Environmental Engineering

Thesis Advisor: Luis E. Vallejo, Professor, Civil and Environmental Engineering Department

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Leanna Seminsky, M.S.

University of Pittsburgh, 2013

Soils containing dispersed and non-dispersed large particles (greater than # 4 sieve) form part of many engineered fills, glacial tills, debris flows, and residual soil deposits. Very little is known about the effect that the large particles have on the shear strength of the soil-large particles mixtures. In this study, the influence of the large particles on the shear strength of the mixtures was evaluated experimentally and numerically. The experimental analysis used direct shear tests on simulated granular materials containing large dispersed particles as well as on real sand-gravel mixtures. The numerical analysis used the Discrete Element Method (DEM). For the dispersed case (the large particles are not in contact), the laboratory and the DEM simulation results indicated that the shear strength of the mixtures increased with the concentration (C) of the large particles in the mixtures. The shear strength of the mixtures with dispersed oversize particles can be obtained from the following relationship: $S_c = S_m (1 + \alpha C)$. In this relationship, S_c is the shear strength of the mixture, S_m is the shear strength of the granular matrix without the oversize particles, C is the concentration by volume of the oversize particles, and α is a constant that varies between 0.4 and 2.5. For the case of the non-dispersed oversize particles (the oversize particles are in contact in the mixture), direct shear tests on sand-gravel mixtures indicated the shear strength of the mixtures can be obtained from the following relationship: $S_c = S_m (1 + 0.7C + 1.8 C^2)$. In general, it was determined that the addition of oversize particles increases the shear strength of the soil in which the oversize particles are either dispersed or non-dispersed.

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1.0 INTRODUCTION

1.1 MOTIVATION AND OBJECTIVES OF THIS STUDY

Soils containing dispersed (non-contiguous) or “floating” large particles (i.e. rock pieces greater in size than # 4 sieve) are common around the world and form part of engineered fills, glacial tills, mudflows, debris flows, solifluction sheets, residual, colluvial and desert soil deposits. In order to obtain the shear strength of these mixtures in the laboratory, large representative samples need to be tested in either the triaxial or the direct shear apparatuses. These large samples require large direct shear or triaxial cells and large loading systems in order to simulate field stress conditions. The use of large triaxial or direct shear equipment makes the tests very time consuming and expensive. The present study is designed to verify a theory of filler reinforcement presented by Guth (1945) that asserts that a mechanical property of a composite made of a mixture of a solid material reinforced by dispersed large particles (**p***) can be obtained from the related mechanical property, **p**, of the matrix, and the concentration by volume of the added dispersed oversized particles, **C**, if the following relationship is used: **$p^* = p\,(1 + 2.5\,C)$**. For this study, **p*** represents the shear strength of a soil-rock mixture, **p** represents the shear strength of the soil matrix, and **C** represents the concentration by volume of the dispersed rock pieces. The validity of this relationship will be evaluated in the laboratory using: (a) plane stress direct shear tests on simulated granular materials containing dispersed large particles, (b) actual direct shear tests on sand with dispersed oversize particles, and (c) by running simulations that make use of Discrete Element Method (DEM). The DEM will be used to simulate the direct shear test on granular materials with dispersed oversized particles. The DEM simulations will be made in two dimensions using Itasca's PFC^{2D} program.

If the relationship advanced by Guth (1945) is found to be feasible for soils with non-contiguous (dispersed) oversized particles, it will be highly beneficial to the geotechnical engineering community in terms of time and money saved since large conventional equipment would not be required to measure the shear strength of soil-rock mixtures (given that the property of the soil matrix, **p**, can easily be measured with conventional geotechnical equipment and the value of **C** can be easily estimated).

The relationship advanced by Guth (1945) applies to sand-gravel mixtures in which the gravel is dispersed in the sand (the gravel particles do not touch) [Fig. 1 (A)]. In this study the relationship advanced by Guth (1945) will be modified to cover situations in which the gravel particles are contiguous (the gravel particles touch each other) in the sand-gravel mixtures as shown in Figs 1(B) and 1(C). The particles of gravel will touch when their concentration increases in the sand-gravel mixtures.

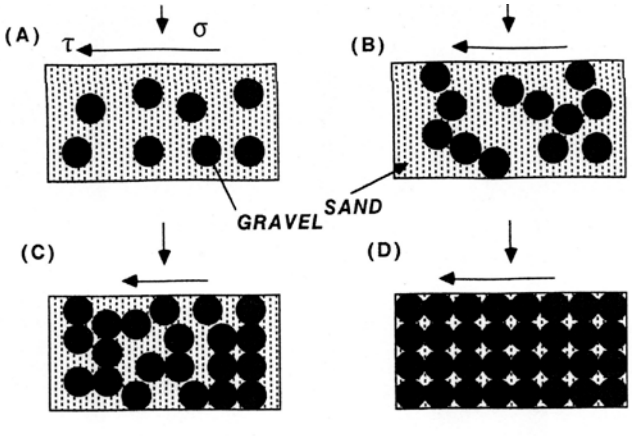


Figure 1. Sand with dispersed (A) and non-dispersed gravel particles (B and C)

1.2 SOILS WITH DISPERSED OVERSIZED PARTICLES

Materials forming part of engineered fills, glacial tills, solifluction sheets, mudflows, debris flows, residual soils, and colluvial and desert soil deposits have a distinct structure which consists of a mixture of a soil matrix (sand, silt, or clay or a combination of these soils) and large particles of gravel or hard clay fragments that are dispersed (fragments do not interact) in the soil matrix. The rock or hard clay fragments are composed of materials larger than the No. 4 sieve (Skempton and Hutchinson, 1969; Magier and Ravina, 1982; Shakoor, et al., 1990; Bolton et al., 1992; Fragaszy et al.,1990, 1992; Poesen and Lavee, 1994; Sowers, 1994; Kropp and Houston, 1994; Taylor, et al., 1995; Budiman, et al., 1995; Noorany and Houston, 1995; Vallejo, 1979, 1980, 1989; Vallejo and Mawby, 2000; Vallejo and Lobo-Guerrero, 2005; Hamidi, et al., 2009) (Figs. 2 to 6). In addition, many soils such as sandy silts and sandy clays (with silt or clay as the soil matrix and sand representing the dispersed particles) can also be considered as mixtures of soils with dispersed oversized particles if one takes into consideration the differences in size between the sand, silt and clay particles (Figs. 6 and 7) (Lafeber, 1966; Morgenstern and Tchalenko, 1967)..

Soil mechanics has dealt mainly with the study of three main soil types: sands, silts, and clays. However, mixtures of soils such as those shown in Figs. 1 through 7 are more commonly found in nature and in earth construction projects than pure sands, silts, and clays. Since the determination of the mechanical properties (i.e. shear strength, hydraulic conductivity, consolidation characteristics) of mixtures such as those depicted in Figs. 1 through 7 has heretofore received scant attention, such an investigation is needed. This study is focused on the determination of the shear strength of granular materials with dispersed and non-dispersed oversized particles [Figs. 1(A), 1(B) and 1(C)].

Figure 2. Colluvial material in a failed slope in Kentucky (Vallejo & Lobo Guerrero, 2005)

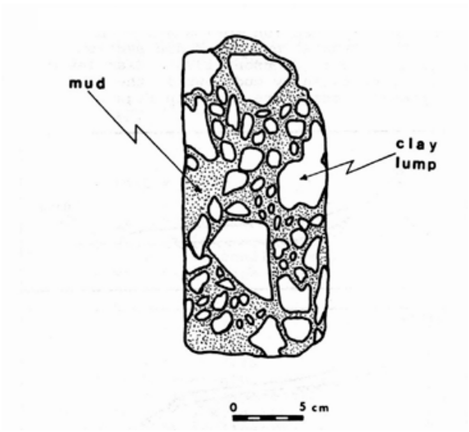


Figure 3. Mudflow fabric (Skempton & Hutchinson, 1969)

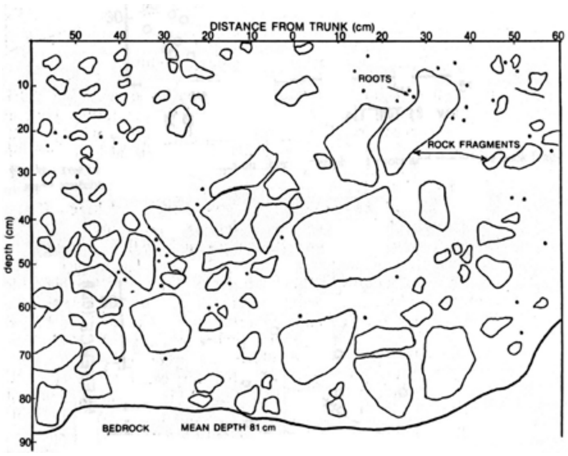


Figure 4. Sand-gravel in a fill at Terra Rossa, Israel (Magier & Ravina, 1982)

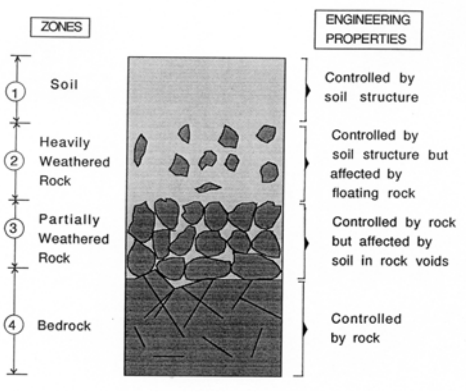


Figure 5. Residual soil-rock deposit due to weathering (Vallejo & Mawby, 2001)

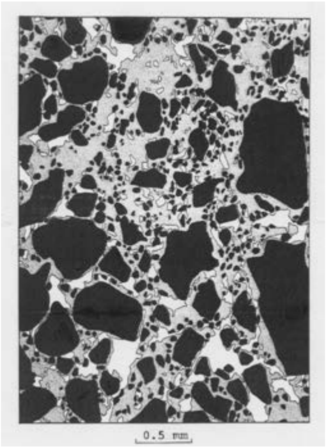


Figure 6. Silt with sand fragments in a soil in Australia (Lafeber, 1966)

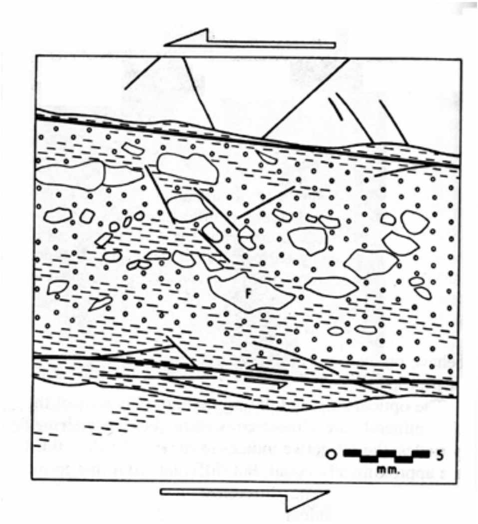


Figure 7. Clay with silt and sand fragments in a shear zone (Morgenstern & Tchalenko, 1967)

1.3 CURRENT KNOWLEDGE ABOUT THE SHEAR STRENGTH OF GRANULAR MATERIALS WITH DISPERSED (NON-CONTIGUOUS) OVERSIZED PARTICLES

Direct shear and triaxial compression tests on granular materials with oversized particles conducted by Holtz and Gibbs (1956); Doddiah, et al. (1969); Vasileva, et al. (1971); Marsal and Fuentes de la Rosa (1976); Fragaszy, et al. (1992); and Vallejo (2001) have indicated that that the shear strength of the mixtures depends upon the relative concentration by *volume* (or weight) of the oversized particles in the mixtures. These researchers found that if the concentration by *volume* was greater than 45% (>70% by weight), the shear strength of the mixtures was basically that of the large particles alone [Figs. 1(C) and 1(D)]. If the concentration by *volume* of the large particles in the mixtures was between 45% and 31 % (70% and 48% by weight), the shear strength of the mixtures was partially controlled by the friction between the large particles [Fig. 1(B)]. If the concentration by *volume* of the large particles was less than 31% (<48% by weight), the shear strength of the mixtures was basically that of the matrix of the smaller particles alone [Fig. 1(A)].

However, triaxial and direct shear strength tests conducted by Bolton et al. (1991), Irfan and Tang (1992), Yagiz (2001), and Vallejo (2001) on mixtures of granular soils with dispersed oversized particles (<31% by *volume*) have indicated that presence of the large particles had a reinforcing effect on the granular matrix that surrounds them. Fig. 7 shows a plot of the direct shear test results on sand-gravel mixtures conducted by Yagiz (2001). The mixtures of sand and gravel were tested in a conventional shear box (65 mm x 65 mm). The sand tested had an average diameter of 0.3 mm and the gravel in the mixture which was angular in shape had an average diameter of 5 mm. This figure clearly indicates that the shear strength of the mixtures increases with the concentration by volume of the gravel in the mixtures. The concentration by volume of gravel in the mixtures tested by Yagiz ranged in value between 0 and 32%. Yagiz (2001), however, does not offer any explanation for the reinforcing mechanisms associated with the presence of the dispersed oversized particles.

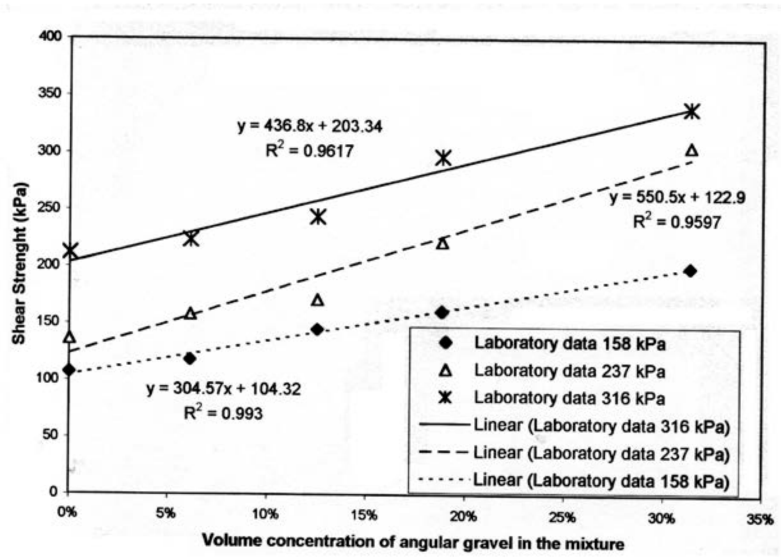


Figure 8. Shear strength of sand-gravel mixtures at various levels on normal stresses in a direct shear test (modified after Yagiz, 2001)

The purpose of this study is to confirm and explain why dispersed (non-contiguous) oversized particles increases the shear strength of the granular matrix in which they “float” or barely touch (Figs. 1 to 6). In addition, an investigation on the feasibility of a method designed to obtain the shear strength of granular mixtures with dispersed oversized particles knowing only the shear strength of the granular matrix and the concentration by volume of the dispersed oversized particles is proposed. If the proposed method is found to work, the shear strength of mixtures such as those depicted in Figs. 2 to 6 will be easily obtained using conventional laboratory tests since one only needs the shear strength of the small soil particles surrounding the oversized particles and the concentration by volume of these oversize particles in order to obtain the shear strength of the composite.

2.0 THE GUTH METHOD TO OBTAIN THE SHEAR STRENGTH OF GRANULAR MATERIALS WITH DISPERSED (NON-CONTIGUOUS) OVERSIZED PARTICLES

It is common practice to improve the mechanical properties of solid materials by reinforcing them through the addition of large rigid particles (Hashin, 1955; Guth, 1945). Thus, mortar (mixture) is prepared by adding sand particles (inclusions) to a cement binder (matrix); rubber is reinforced by fillers consisting of small dust like particles which may be assumed to be infinitely rigid in comparison with rubber (Guth, 1945). It has been shown by Guth (1945) that a mechanical property, **p*** (such as viscosity, Young's modulus of elasticity, shear modulus, thermal conductivity or dielectric constant), of these mixtures can be obtained from the related mechanical property, **p**, of the matrix, and the concentration by volume of the added particles, **C**, if the following equation is used:

$$p^* = p\,(1 + 2.5\,C) \qquad (1)$$

Eq. (1) was obtained by Guth (1945) using the hydrodynamic analysis of two-phase suspension systems developed by Einstein (1906). Arnstein and Reiner (1945) found that the viscosity of a mixture of a cement matrix and sand particles (property p*) can be obtained from the viscosity of the cement matrix (property p) by using Eq. (5) for suspensions having sand concentrations (C) as large as 0.6.

Eq. (1) seems to be very general and could be applied to the case of the shear strength of granular materials with dispersed oversized particles. For this case the following relationship seems appropriate,

$$S_c = S_m (1 + 2.5 C) \tag{2}$$

In Eq. (2), $\mathbf{S_c}$ is the shear strength of a granular material with dispersed oversized particles, $\mathbf{S_m}$ is the shear strength of the granular matrix in which the large particles are dispersed, and **C** is the concentration by volume of the oversized particles.

One of the objectives of this study was to confirm and explain the validity of Eq. (2) using plane stress direct shear tests on simulated granular mixtures (Vallejo, 1987; Vallejo, 1991), direct shear tests on sand-gravel mixtures, and Discrete Element Method simulations in two dimensions of the direct shear tests on granular materials with dispersed oversized particles.

2.1 APPLICATION OF THE GUTH METHOD TO THE RESULTS OBTAINED BY YAGIZ, 2001

Very few tests designed to obtain the shear strength of granular materials with dispersed oversized particles have been conducted to date. The reason for the lack of test results in these materials has to do with the fact that large representative samples need to be tested in either the triaxial or the direct shear apparatuses. These large samples require large direct shear or triaxial cells and large loading systems in order to simulate field stress conditions. The use of large triaxial or direct shear equipment makes the tests very time consuming and expensive. To solve this, Yagiz (2001) conducted some conventional direct shear tests using a 65 mm shear box on mixtures of sand and gravel (the sand had an average diameter of 0.3 mm and the gravel with an angular shape had a diameter equal to 5 mm) (Fig. 8). Using the values of the shear strength of the sand alone, $\mathbf{S_m}$, as well as the concentration by volume, **C**, of the gravel in the mixtures, the values of the shear strengths of the mixtures, $\mathbf{S_c}$, were evaluated using the Guth's model represented by Eq. (6) (Guth, 1945). The values of the shear strength, $\mathbf{S_c}$, of the sand-clay mixture obtained using the Guth's model [Guth, 1945; Eq. (2)] were superimposed on the values obtained by Yagiz (2001) (Fig. 9). Fig. 9 shows that the Guth's model predicts very well the shear strength of the mixtures. Thus, Eq. (2) seems to predict reasonably well the shear strength of granular material with oversized particles. However, the direct shear tests conducted by Yagiz (2001) were very limited. The tests by Yagiz (2001) did not consider: (a) the effect of the size of the shear box and that of the particles tested, (b) the effect of degree of roughness of the oversized particles, and (c) the effect that the grains forming the matrix have on the shear strength of the mixtures. Also, Yagiz (2001) does not offer any explanation for the reinforcing mechanisms associated with the presence of the dispersed oversized particles.

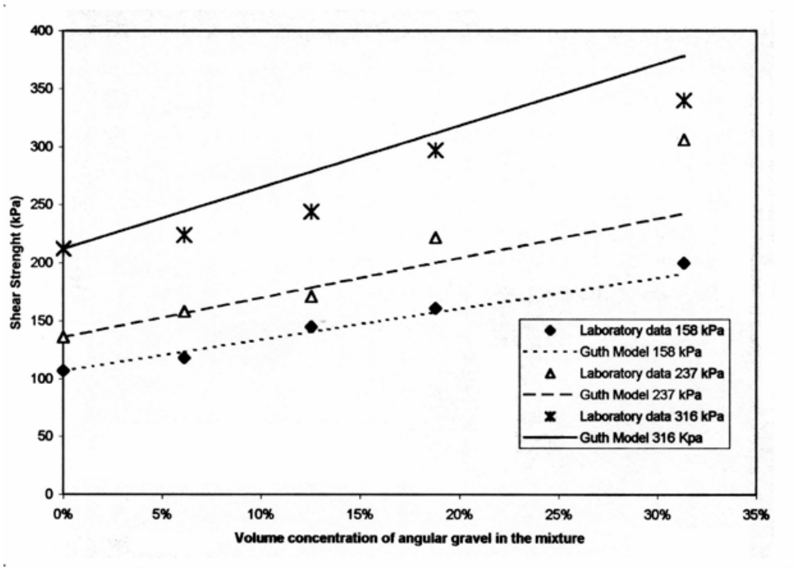


Figure 9. Comparison of laboratory results of Yagiz (2001) with the Guth's model (Eq. 2)

Unless a detailed explanation of the reinforcing mechanisms associated with the dispersed oversized particles is available, the general application of Eq. (2) cannot be fully accepted. One of the purposes of this study is to confirm and explain why dispersed oversized particles increase the shear strength of the granular matrix in which they "float." The confirmation and validation of Eq. (2) will be done by using: (a) plane stress direct shear tests on simulated granular materials containing dispersed large particles, (b) actual direct shear tests on sand with dispersed oversize particles, and (c) simulations that make use of Discrete Element Method (DEM).

2.2 THE IMPORTANCE OF THE RESULTS TO ENGINEERING PRACTICE

Soils containing dispersed or “floating” large particles (i.e. rock pieces greater in size than # 4 sieve) are common around the world and form part of engineered fills, glacial tills, mudflows, debris flows, solifluction sheets, residual, colluvial and desert soil deposits. In order to obtain the shear strength of these mixtures in the laboratory, large representative samples need to be tested in either the triaxial or the direct shear apparatuses. These large samples require large direct shear or triaxial cells and large loading systems in order to simulate field stress conditions. The use of large triaxial or direct shear equipment makes the tests very time consuming and expensive. The present study is designed to verify a theory of filler reinforcement presented by Guth (1945) that asserts that a mechanical property of a composite made of a mixture of a solid material reinforced by dispersed large particles (**p***) can be obtained from the related mechanical property, **p**, of the matrix, and the concentration by volume of the added particles, $\mathbf{C_f}$, if the following relationship is used: $\mathbf{p^* = p\ (1 + 2.5\ C_f)}$. For our proposed study, **p*** represents the shear strength of a soil-rock mixture, **p** represents the shear strength of the soil matrix, and $\mathbf{C_f}$ represents the concentration by volume of the dispersed rock pieces. The validity of this relationship will be evaluated using laboratory tests and a numerical analysis that makes use of the Discrete Element Method (DEM)

If the relationship advanced by Guth (1945) is found to be feasible for soils with dispersed oversized particles, it will be highly beneficial to the geotechnical engineering community in terms of time and money saved since large conventional equipment would not be required to measure the shear strength of soil-rock mixtures (given that the property of the soil matrix, **p**, can easily be measured with conventional geotechnical equipment and the value of **C** can be easily estimated).

3.0 THE PLANE STRESS DIRECT SHEAR TESTS ON SIMULATED GRANULAR MATERIALS WITH DISPERSED (NON-CONTIGUOUS) OVERSIZED PARTICLES

For the purpose of understanding the mechanisms involved in the shear strength of granular materials with dispersed large particles, an open face, two-dimensional direct shear apparatus was used (Fig. 10). This apparatus is called the Plane Stress Direct Shear Apparatus (PSDSA) (Vallejo, 1991). The granular matrix will be simulated by a mixture of wooden sticks having polygons as their cross sectional areas. These polygons resemble the profiles of actual granular materials (Fig. 4).

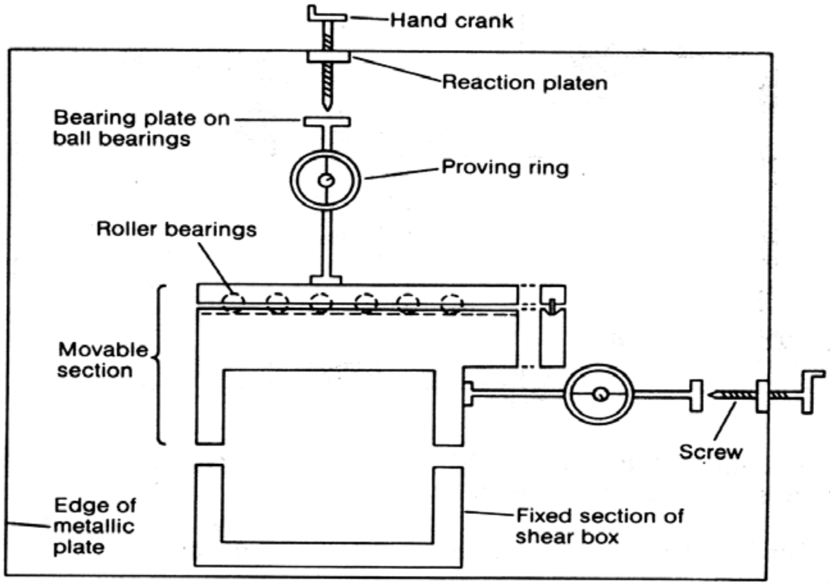
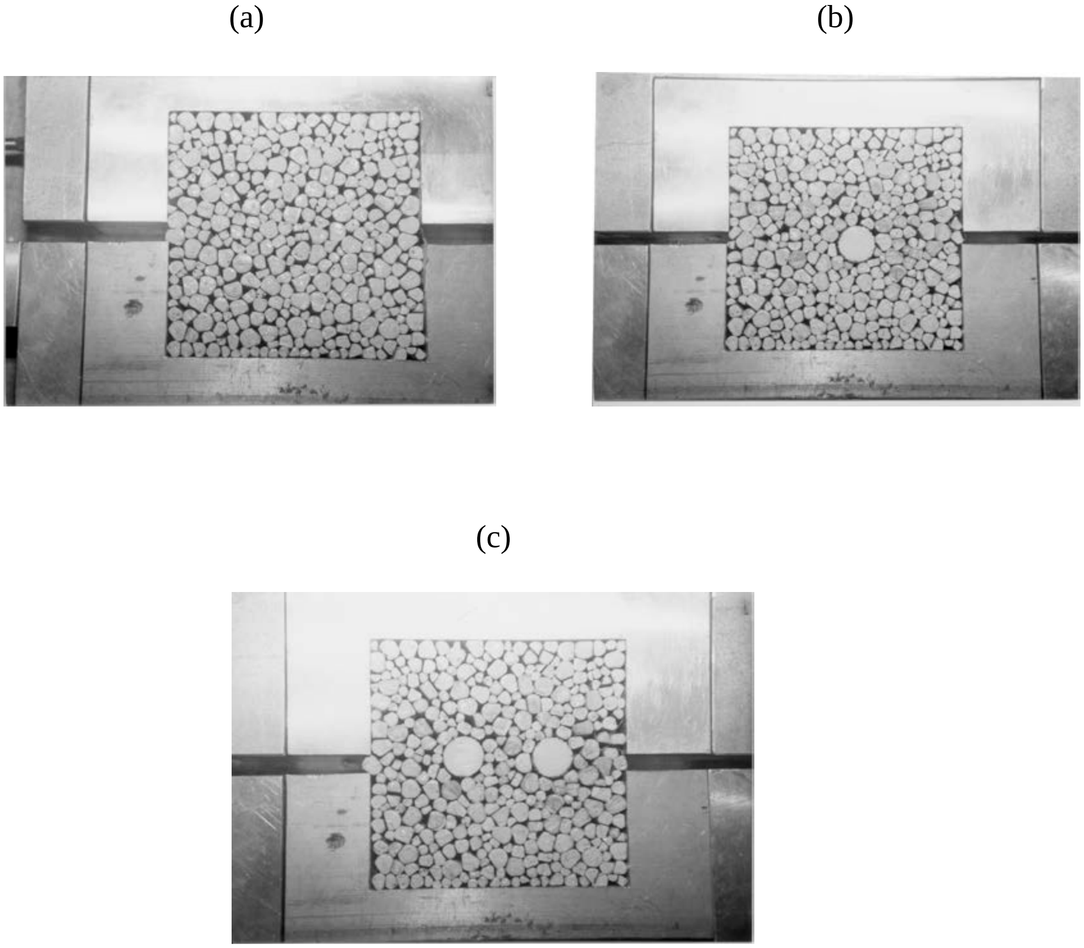


Figure 10. The Plane Stress Direct Shear Apparatus (Vallejo, 1987, 1991)

The wooden sticks forming the granular matrix will have 3 different average diameters. These will be equal to 6, 4, and 2.7 mm. Thus, the granular matrix as a whole will be made up of sticks having an average diameter equal to 4.2 mm. The oversized large particles will be simulated by rough circular cylinders with a diameter equal to 12 mm. The irregular sticks as well as the circular cylinders have a length equal to 25 mm. The mixture of wooden sticks and cylinders were placed inside two U forms that comprise the box in the Plane Stress Direct Shear Apparatus (PSDSA) (Figs. 10 and 11). The area inside the two U forms is a square area with sides measuring 7.6 cm in length. The open face of the shear apparatus formed by the two U forms allows the recording of the changes taking place in the mixture during shearing. Two proving rings measure the normal and shear forces applied to the mixtures. Dial gauges measure the normal and shear displacements. The changes in fabric experienced by the mixture as well as the interaction between the granular matrix and the large particles during shear was recorded using digital photographs of the open face of the PSDSA.



(a)

(b)

(c)

Figure 11. Simulated granular mixture in the PSDSA before shear testing

3.1 DIRECT SHEAR TESTING IN THE PSDSA

The simulated granular mixtures depicted in Fig. 11 were subjected to shear in the PSDSA. The shears of the mixtures were carried out using two normal stresses. These were equal to 99.6 and 199.3 kPa. The rate of shearing of the mixtures was equal to 2mm/min. Fig. 12

shows the shear stress versus the horizontal displacement relationships for samples containing the matrix alone. The samples have one and two 12 mm diameter cylinders which represent the large particles (Fig.11). Fig. 13 represents the vertical displacement versus horizontal deformation in the PSDSA testing.

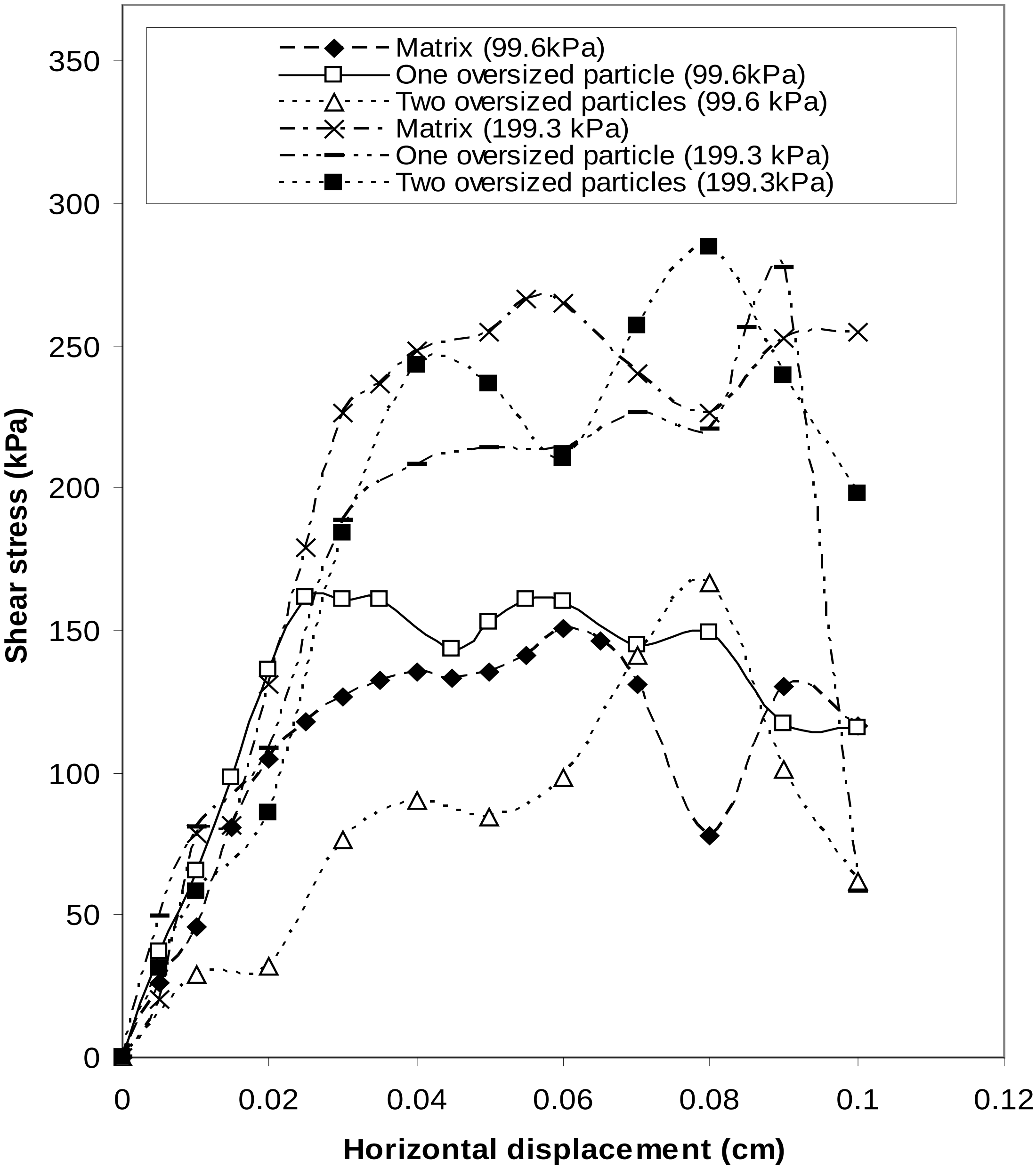


Figure 12. Shear stress versus horizontal displacement for the mixtures of Fig. 11

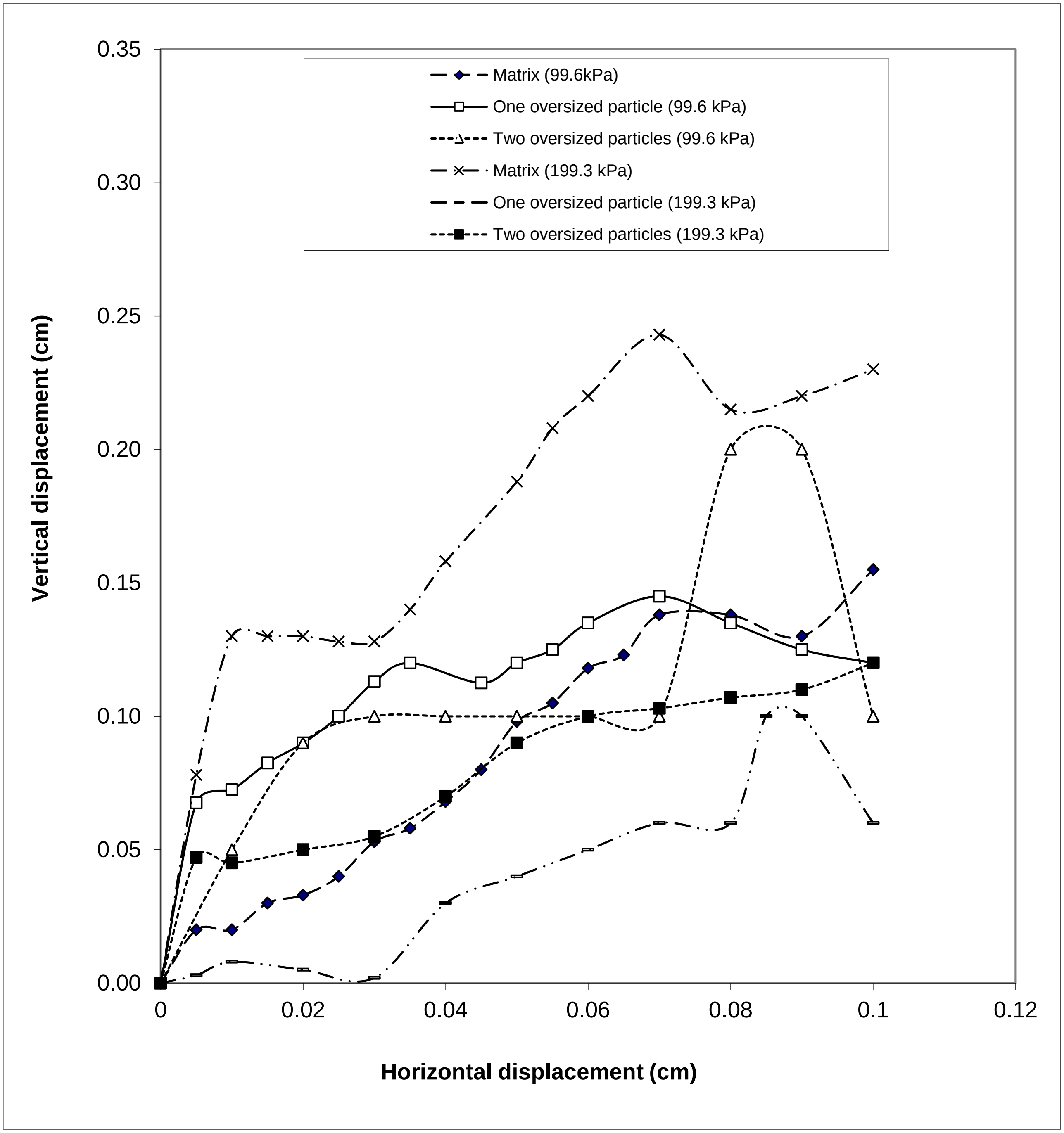


Figure 13. Vertical deformation versus displacement in the PSDSA testing

The peak values of the shear stress plots of Fig. 12 have been used to plot the shear strength versus the area concentration of the large cylinders in the sample. This area concentration is equal to the cross sectional area of the large cylinders in the mixture divided by the area of the whole mixture (7.62 cm x 7.62 cm) (Fig. 3). The resulting plot is shown in Fig.14. This figure shows that the shear strength of the mixture increases as the number of large cylinders increases in the mixture. An equation that represents this increase is of the form

$$S_c = S_m (1 + 2\ Ca) \qquad (3)$$

where S_c is the shear strength of the mixture, S_m is the shear strength of the matrix, and Ca is the area concentration of the large cylinders in the mixture. The results of Fig. 14 and Eq. (3) indicate that the overall shear strength of the simulated granular mixtures increases with an increase in the number of the large cylinders. Thus, in the case of real sand-gravel mixtures, it is expected that the shear strength of these mixtures will increase with the volume concentration of the gravel in the mixtures.

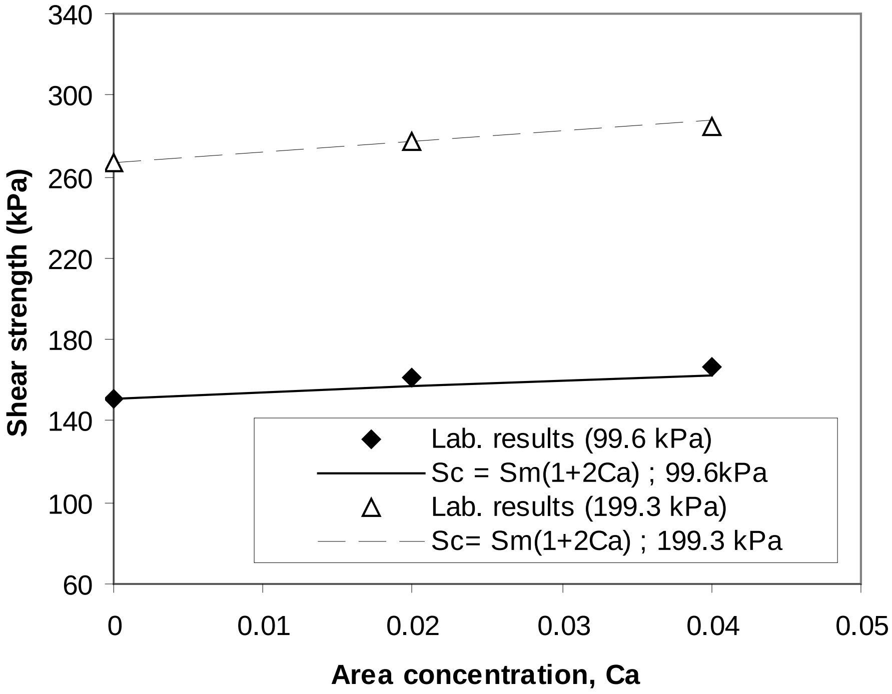


Figure 14. Shear strength of the simulated granular mixtures in function of the area concentration of the large cylinders in the mixture.

4.0 DIRECT SHEAR TESTS ON SAND-GRAVEL MIXTURES WITH DISPERSED OVERSIZED PARTICLES

The validity of the Guth (1945) method was investigated on mixtures of sand and gravel. The gravel used had an average diameter d_{50} = 6 mm, a specific gravity G_s = 2.40 and a coefficient of uniformity C_u =1.9. The sand used was an Ottawa sand that had an average diameter d_{50} = 0.59 mm, a specific gravity G_s = 2.65 and a coefficient of uniformity C_u =1.3. Mixtures of sand and gravel were placed in a plastic bag and shaken until they appeared visually homogeneous. The weight of the mixtures was kept constant while equaling 600 grams. The samples tested have different percentages by weight of sand and gravel. The percentages by weight of the gravel in the mixtures were converted to percentages by volume C (volume of gravel in the mixture/volume of the total mixture) using the following relationship advanced by Agarwal and Broutman (1979),

$$C = (\gamma_c/\gamma_p)\, C_w \tag{4}$$

where γ_c is the unit weight of the composite, γ_p is the unit weight of the rigid dispersed particles, and C_w is the concentration by weight of the rigid particles.

The mixtures from the plastic bag were slowly poured into the direct shear apparatus. The direct shear apparatus had a box in which circular samples can be tested. The diameter of the samples that could be tested was equal to 6.2 cm. The samples in the direct shear apparatus were tested under three normal stresses that were equal to 116.92, 233.85, and 350.77 kPa. The results of the tests in the form of shear strength versus horizontal displacement are shown in Figs. 14, 15 and 16.

4.1 THE SHEAR STRENGTH OF THE MIXTURES CONTAINING DISPERSED GRAVEL

Using the results depicted by Figs. 15, 16, and 17, one can plot the relationships between the peak shear strength measured in the direct shear test and the concentration by volume, C, of the gravel in the mixtures. This has been done in Fig. 18 for the case in which the gravel is dispersed in the mixture (C < 30%) [Fig. 1 (A)].

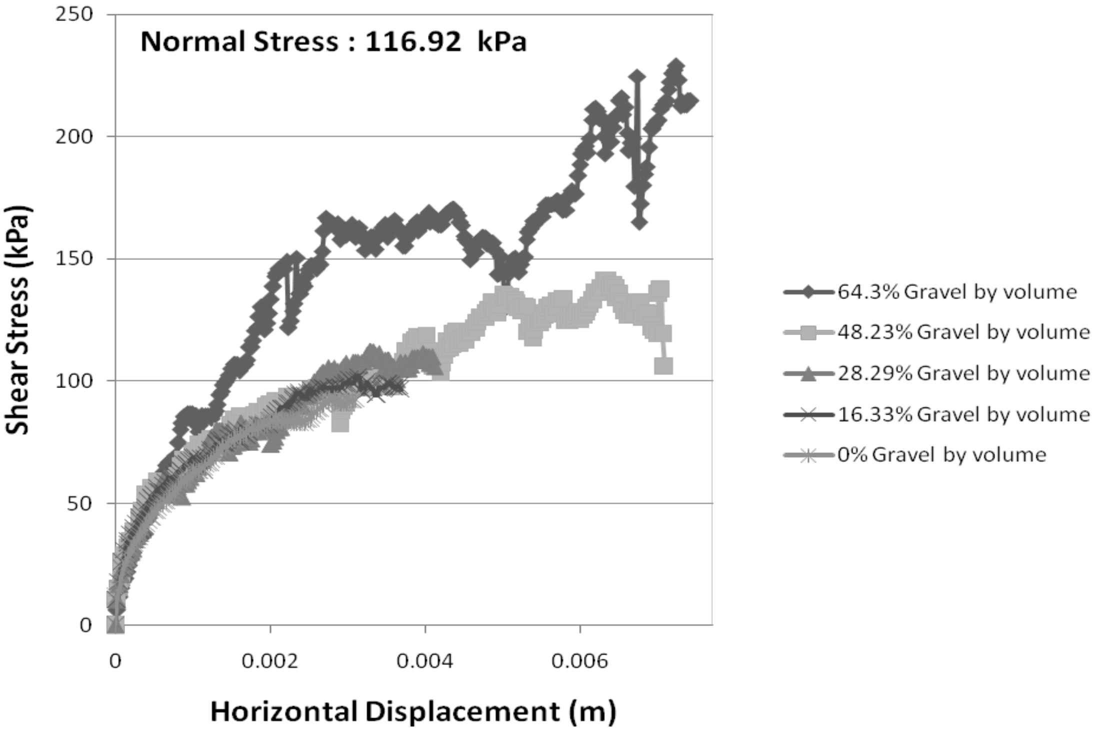


Figure 15. Shear stress versus horizontal displacement for the case in which the normal stress is 116.92 kPa and for different concentrations by volume of the gravel in the samples.

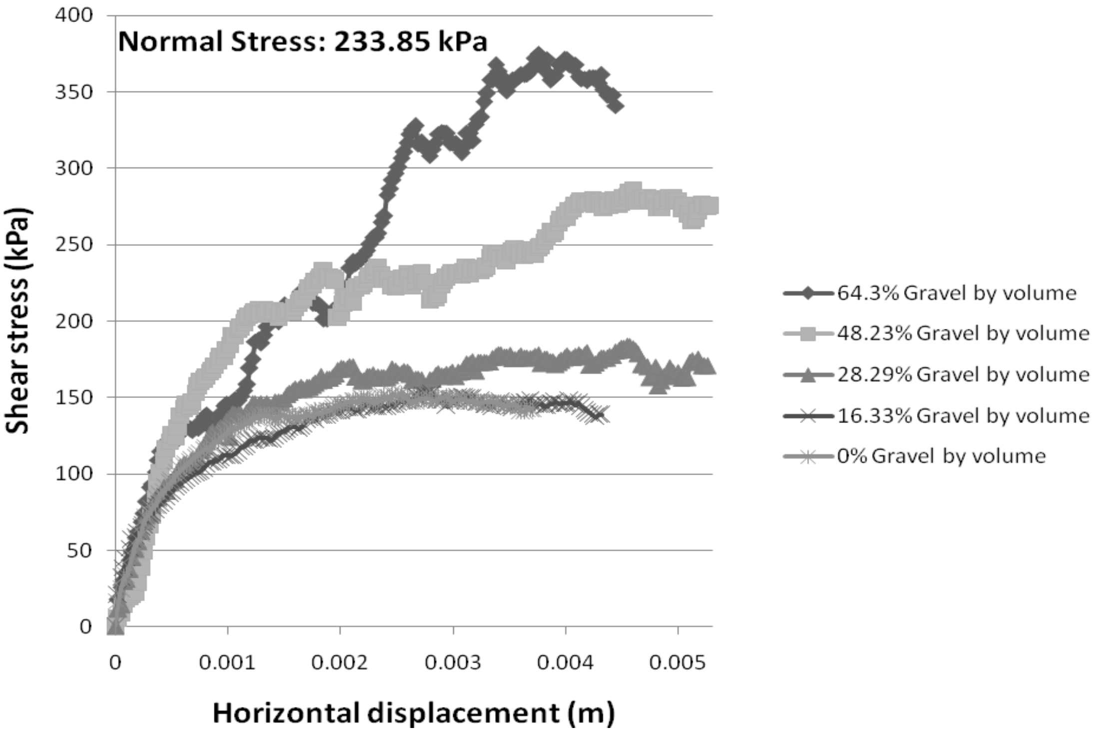


Figure 16. Shear stress versus horizontal displacement for the case in which the normal stress is 233.85 kPa and for different concentrations by volume of the gravel in the samples.

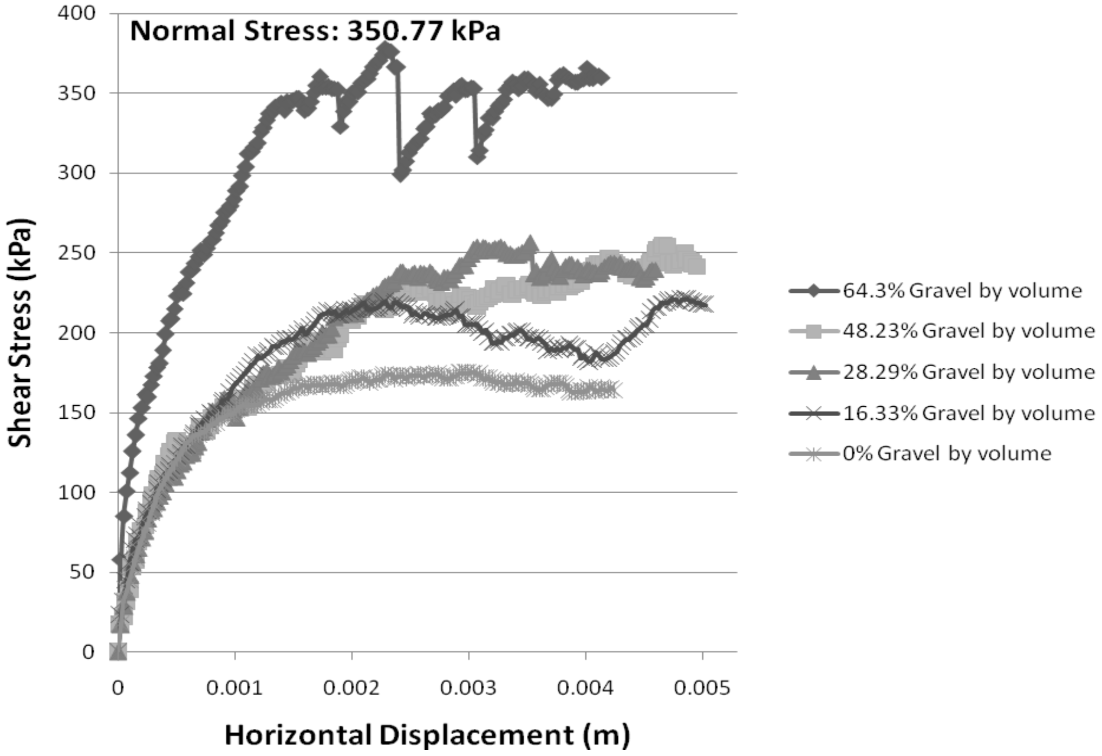


Figure 17. Shear stress versus horizontal displacement for the case in which the normal stress is 350.77 kPa and for different concentrations by volume of the gravel in the samples.

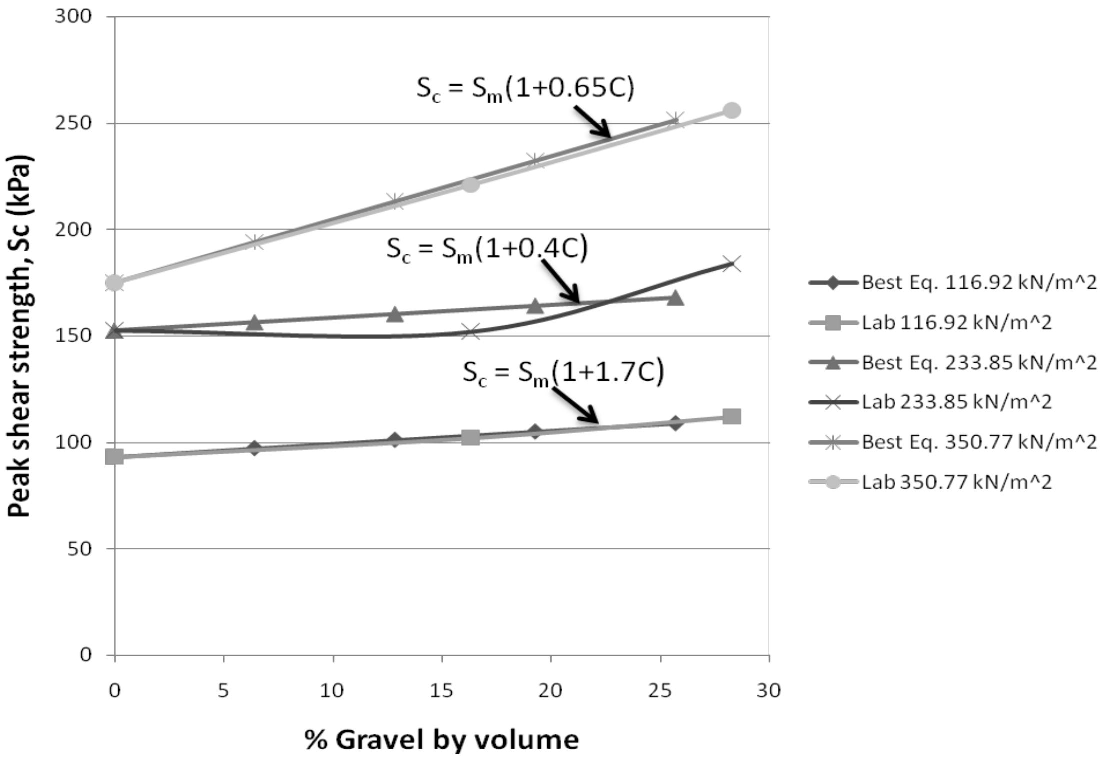


Figure 18. Shear strength of the mixture for volume concentration of gravel $C < 30\%$ (dispersed case).

An analysis of Fig. 18 indicates that the relationships that best fit the laboratory results are not similar to the one advanced by Guth (1945) (Eq. 2). The direct shear test results are similar to the ones obtained in the PSDSA apparatus (Eq. 3). The only variation between the Guth (1945) relationship (Eq. 2) and the ones obtained using the PSDSA tests and the direct shear tests is in the constant multiplying the volume concentration value C.

5.0 THE DISCRETE ELEMENT METHOD TO OBTAIN THE SHEAR STRENGTH OF GRANULAR MATERIALS WITH DISPERSED (NON-CONTIGUOUS) OVERSIZED PARTICLES

Next, the Discrete Element Method will be used to analyze the shear strength of simulated sand-gravel mixtures. The DEM simulations will use the geometry of the mixtures used for the PSDSA tests (Fig. 11).

5.1 CONFIGURATION OF THE SAMPLES

The PFC^{2D} program produced by Itasca (Itasca Consulting Group Inc., 2002) was used for the simulation of the direct shear tests on granular material with dispersed oversized particles. The first step to the configuration of the sample was the construction of the shear box. The box had two sections each with a width of 6 cm and a height of 1.5 cm. The two sections were placed on top of each other and after the circular particles were generated inside the box, the gap between the two sections was maintained at 0.5 mm. The depth of the sample was assumed to be equal to 1 m. The shear and normal stiffness of the walls forming the box were set to 1×10^9 N/m. The coefficient of friction between the circular particles and the particles and the walls was set to 0.7.

After the construction of the box, 1000 particles representing the granular matrix and having a diameter of 0.63 mm were generated inside the box. The density of the particles was set to 2,500 kg/m^3 and their normal stiffness and shear stiffness were set to 1x10^8 N/m. Their positions were randomly chosen by the program, having the limitation of no overlap between particles. A normal gravity field (9.8 cm/sec^2) was used during the simulation. In order to simulate the dispersed oversized particles, 52 particles of diameter equal to 0.63 mm were removed and replaced by an oversized particle measuring 5 mm. If an additional oversized particle was needed to be placed in the sample, the same number of smaller particles were removed and replaced by another large particle with a 5 mm diameter (Fig. 19). The tests were run under a constant normal compressive load equal to 2x10^4 N. After the normal compressive force was applied to the sample, the shearing started by moving the upper section of the shear box to the left with a constant velocity of 0.44 mm/sec. The tests ended when the horizontal displacement was equal to 5 mm. Also, using a subroutine available in the PFC^{2D} code, one can obtain the value of the shear stress in function of the horizontal deformation. In this study, the peak shear resistance that was measured in the simulation represents the shear strength of the mixture.

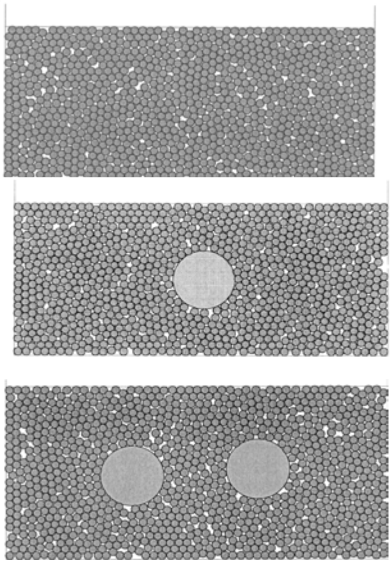


Figure 19. Simulated samples by DEM containing zero, one and two large cylinders

5.2 RESULTS OF THE SIMULATIONS

The DEM simulations of the direct shear tests were carried out on mixtures having zero, one, and two oversized particles. Fig. 20 shows typical DEM results for the samples with zero, one and three oversized particles. These figures show the force chains and their intensity (the thicker the force chains, the bigger the force chain value with their maximum values shown at the top of the figures) for the samples with 3.5 mm of horizontal displacement.

An analysis of Fig. 20 indicates that the larger force chains which were compressive in nature were directed toward the large particles and were transmitted to them by the smaller surrounding particles. When the horizontal displacement in the simulated test reached a 3.5 mm value, the force chains were inclined at about 45 and 135 degrees with respect to the horizontal axis of the cross sectional area of the large particles.

It is usually assumed that when samples of granular materials with oversized particles are subjected to either compressive or direct shear stress conditions, the smaller particles in the mixture distribute the loads uniformly around the perimeter of the bigger particles. This uniform load distribution produces low compressive stresses on the bigger particles which allows them to survive without breakage (Sammis, 1997). The results shown in Fig. 20 indicate that this is not the case. Under direct shear, the smaller particles concentrate large compressive forces that are exerted on a small section of the perimeter of the large particles. These high concentrated compressive forces exerted by the smaller particles on the large particles have also been found to be effective by Cheng and Minh (2009) during the shearing of poly-disperse granular materials.

The peak shear stress values obtained during the shearing of the mixture shown in Figs. 19 and 20 were plotted against the area concentration of the large cylinders in the mixture. The result of the plot is shown in Fig. 21.

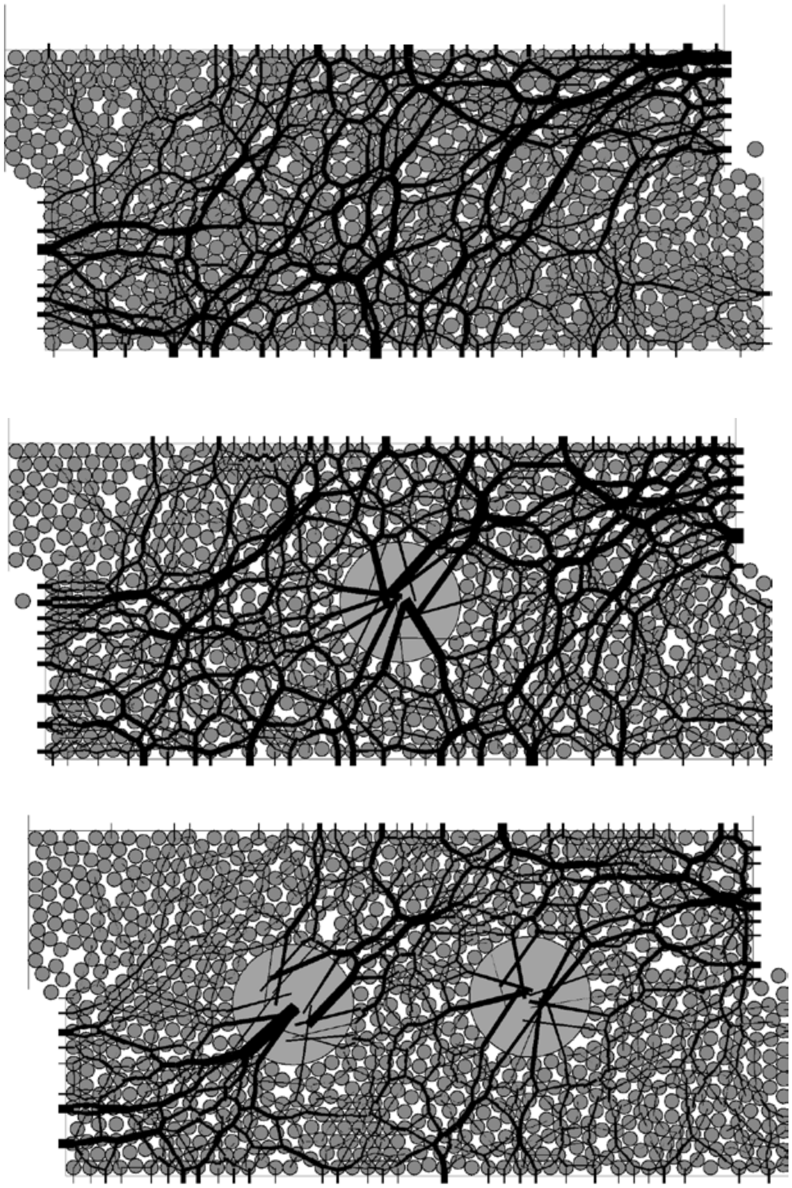


Figure 20. Force chains in samples with 0, 1 and 2 large particles at a horizontal shear displacement equal to 3.5 mm.

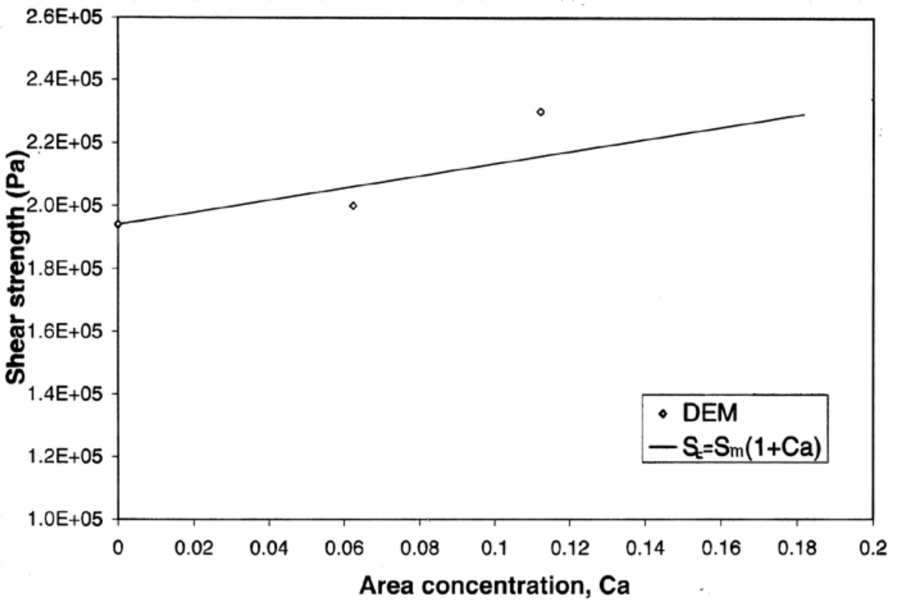


Figure 21. Shear strength versus the area concentration of the large cylinders in the simulated granular mixture

An analysis of Fig. 21 indicates that the presence of the large cylinders in the mixture has a reinforcing effect. That is, as the number of large cylinders increases in the mixture, its shear strength also increases. The best fit line shown in Fig. 21 has an equation of the form:

$$S_c = S_m (1 + Ca) \qquad (5)$$

which is very similar to Eq. (2). It should be noted that the DEM simulations did not represent exactly the shape of the particles forming part of the laboratory experiments. Also, the sizes of the particles used in the PSDSA experiments were different than those used in the DEM simulations. However, the general results of the laboratory tests are corroborated by the DEM simulations. In addition, the DEM simulations help to explain the results obtained in the PSDSA tests and the direct shear tests on sand-gravel mixtures. That is, the shear strength of granular materials with dispersed oversized particles is increased with the concentration by volume of the oversized particles in the mixture.

6.0 ANALYSIS OF THE LABORATORY AND NUMERICAL RESULTS FOR THE CASE OF GRANULAR MATERIALS WITH DISPERSED OVERSIZED PARTICLES

The laboratory and numerical results of the shear strength of granular materials with dispersed oversized particles ($C < 30\%$) have indicated that the shear strength of the mixtures is improved by the addition of oversized particles. From the laboratory and numerical analyses it was determined that an equation of the following form works best,

$$S_c = S_m (1 + \alpha C) \qquad (6)$$

where α is a constant that varies between 0.4 and 2.5. The above equation is similar to the Guth (1945) (Eq. 2). The only difference between Eq. (6) and Eq. (2) is the constant α that was found to vary between 0.4 and 2.5 (Figs. 9, 14, 18, and 21).

7.0 ANALYSIS OF THE DIRECT SHEAR TEST RESULTS FOR THE CASE OF GRANULAR MATERIALS WITH NON-DISPERSED OVERSIZED PARTICLES

In the present study, direct shear tests on sand-gravel mixtures were conducted for mixtures in which the gravel was non-dispersed ($C > 30\%$). The samples reflect the samples shown in Figs. 1(B) and 1(C). In these samples, the gravel particles make contact. For this case, Guth (1945) also stated that an equation of the following form could determine the shear strength of the mixture,

$$S_c = S_m (1 + \alpha C + \beta C^2) \qquad (7)$$

where α and β are constants to be determined. Also, the βC^2 term takes into consideration the contact effect of the gravel in the overall shear strength S_c.

The direct shear tests on mixtures of sand and gravel depicted in Figs. 15, 16 and 17 were used to establish the values of α and β and the validity of Eq. (7) to obtain the shear strength of granular materials with non-dispersed oversized particles. Fig. 22 shows the results of the shear strength of sand with gravel content in excess of 30%.

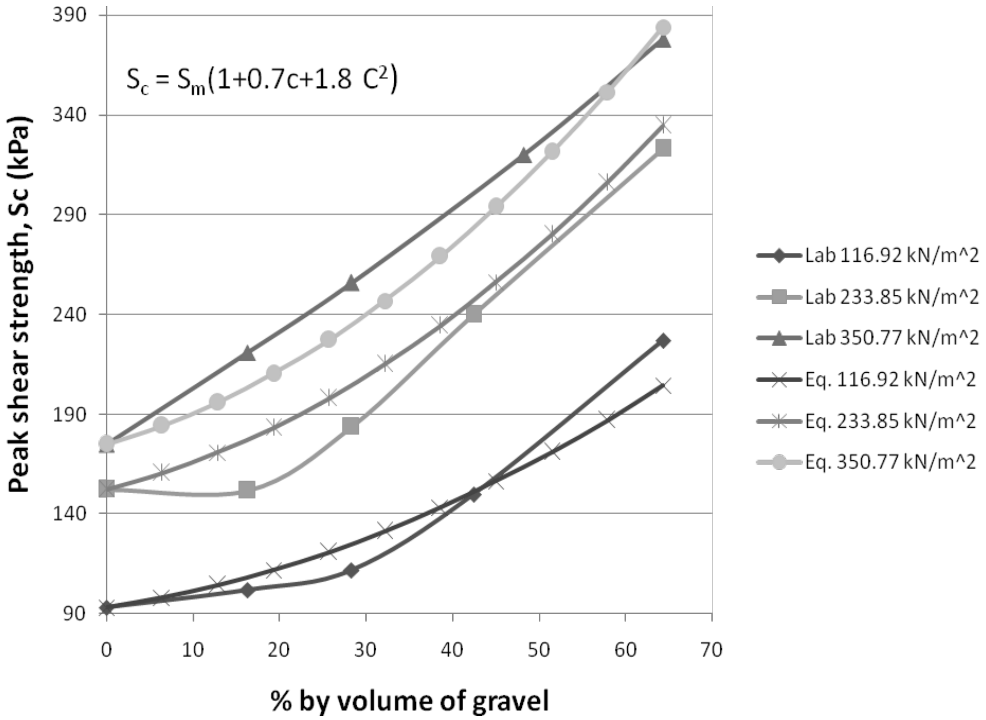


Figure 22. Shear strength of sand-grave mixtures (S_c) versus the volume concentration of the gravel (C) for different values of the normal stress in the direct shear tests.

An analysis of Fig. 22 indicates that a single equation can be used to calculate the shear strength of the sand-gravel mixtures for different values of the normal stress in the direct shear tests. This equation is,

$$S_c = S_m (1 + 0.7C + 1.8C^2) \qquad (8)$$

In Eq. (8), the values of α and β are equal to 0.7 and 1.8 respectively. The correlations coefficients (R^2) were found to be greater than 0.9 for the three tests that used a different value of the normal stress in the direct shear tests. These good correlations coefficients (0.99, 0.931, and 0.938) indicate that Equation (8) can be used with confidence in the calculation of the shear strength of the mixtures tested. Also, Fig. 22 indicates that the addition of gravel to sands improves the shear strength of the resulting mixture.

8.0 CONCLUSIONS

In the present study, the shear strength of granular materials with dispersed oversized particles (large particles are not in contact) and non-dispersed oversized particles (large particles are in contact) were analyzed using direct shear tests on actual sand-gravel mixtures and on simulated granular materials. The simulated granular materials in the form of wooden cylinders were tested in the Plane Stress Direct Shear Apparatus. The Discrete Element Method was used to analyze the shear strength of circular particles with non-dispersed large circular particles. From the laboratory and numerical analyses the following conclusions can be reached:

1. For the dispersed and non-dispersed cases, the oversized particles had a reinforcement effect on the mixtures and their presence caused an increase in their shear strength. The larger the concentration of the oversize particles, the larger was the measured shear strength.

2. For the dispersed case, the shear strength of the mixtures, S_c, can be obtained from the shear strength of the granular matrix, Sm, and the concentration by volume of the oversize particles by using an equation of the following form: $Sc = S_m (1 + \alpha C)$. The constant α varies between 0.4 and 2.5. This variation depends on the type of materials tested and the laboratory equipment used in the shear strength testing.

3. For the non-dispersed case, the shear strength of sand-gravel mixtures can be obtained from the following relationship: $S_c = S_m (1+ 0.7 C + 1.8 C^2)$. This equation is valid regardless of the value of the normal stress acting on the mixtures.

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